EFFECTS OF THE OCEAN TIDE TRANSMITTED THROUGH THE CONFINED AQUIFER

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BY

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1. Introduction

The analytical method for the unsteady flow of artesian water is based on the assumptions for the compressible properties of both the aquifer column and its confined water. In order to develop the applicability of such assumptions, it is desirable to gain more accurate informations for the elastic compression of the aquifer not only with the indirect verification by analysing the observed transmission of pressure disturbance through the aquifer but with the direct measurements for the change in the thickness of the aquifer according to that in the pressure of confined water.

One of other important problems in geohydrology under artesian condition is to find the theoretical method to analyse the variation in the piezometric level, according to that in external load acting on the aquifer as a function of time and position. Jacob's expression [1940] for tidal or barometric efficiency is not sufficiently applied to such problems, because it is only applied to a special case when the variation in the loading weight acts uniformly on the whole surface of the aquifer.

This paper includes the summary of studies by the author concerned with the above-mentioned problems, some parts of which were separately published in these eight years in the Japanese Journal of Limnology. [Kikkawa, 1955, 1956, 1959]

2. Fundamental equation

Let us consider one-dimensional flow through a homogeneous confined aquifer, which is expressed in general form as follows.

$$q=-T\frac{\partial h}{\partial x} \tag{1}$$

$$\frac{\partial(\rho q)}{\partial x}=-\frac{\partial(\rho\alpha D)}{\partial t}=-\rho\frac{\partial(D\alpha)}{\partial t}-D\alpha\frac{\partial\rho}{\partial t} \tag{2}$$

where h is the level of piezometric surface, q is the amount of flow along the direction of x through unit width of the aquifer, α is porosity, D is thickness of the aquifer, T is the coefficient of transmissibility and ρ is density of the water. Assuming that grains of porous media are rigid and the volume change in the skeleton of the aquifer or the confined groundwater is proportional to change in pressure acting on it, change of external load represented by δp is considered to be born in part, $\delta(p-h)$, by the skeletal aquifer and in the other part, δh by the confined water so as to satisfy the next relations.

$$\delta(D\alpha)=\delta D=-\frac{D}{E_s}\delta(p-h)$$

$$\delta\rho=\frac{\rho}{E_w}\delta h$$

where E_s or E_w is the modulus of compressibility of skeletal aquifer or water, and p is expressed by the height of water column. Inserting above relations to equ. (2) and neglecting the derivative of ρ by x,

$$\frac{\partial q}{\partial x}=\frac{D}{E_s}\frac{\partial(p-h)}{\partial t}-\frac{D\alpha}{E_w}\frac{\partial h}{\partial t}$$

Combining the above equation of continuity with equ. (1), we can find the fundamental equation governing the unsteady flow under any variation in external load.

$$\left.\begin{aligned}&\frac{\partial(h-\theta p)}{\partial t}=\frac{T}{S}\frac{\partial^2 h}{\partial x^2}\\&S=D\left(\frac{1}{E_s}+\frac{\alpha}{E_w}\right),\ S'=\frac{D}{E_s}\\&\theta=\frac{S'}{S}=\frac{E_w}{E_w+\alpha E_s}\end{aligned}\right\}\qquad(3)$$

When variations in p with time are uniform all over the aquifer, it follows from the above equation that

$$\delta h=\theta\delta p$$

Thus, the coefficients θ and $(1-\theta)$ are respectively shown to correspond with the Jacob's tidal and barometric efficiencies.

3. Fluctuations in the piezometric level produced by those in the loading weight upon the aquifer

It is well-known that the tidal fluctuations in the piezometric level in semi-infinite aquifer in contact with the ocean can be analysed by the next formula.

$$h=h_0 e^{-\sqrt{\frac{\omega S}{2T}}x}\sin\left(\omega t-\sqrt{\frac{\omega S}{2T}}x\right)\qquad(4)$$

where h_0 is the amplitude of the ocean tide, ω is angular velocity for a tidal period and x is the distance inland from the submarine outcrop. For the purpose of illustrating the applicability of equ. (3) in the last section, let us treat the effect of ocean tide on the coastal semi-infinite aquifer having no suboutcrop on the sea bottom. Hereafter, h and p in equ. (3) are taken as net rises with reference to their mean stages.

Assuming that the ocean tide acts only as the fluctuations in the loading weight upon the submarine part of the aquifer from the closed boundary at $x=-a$ to the seashore at $x=0$, p is represented as follows.

$$0 \geqq x \geqq -a \quad : \quad p = h_0 \sin \omega t$$
$$x > 0 \quad : \quad p = 0$$

Since no current can be occured through the boundary,

$$\frac{\partial h}{\partial x} = 0 \quad \text{at } x = -a$$

Mathematical development of equ. (3) under the abovementioned conditions leads the following final solution satisfied in the inland region $x \geqq 0$.

$$\left.\begin{aligned} h &= \frac{\theta}{2} h_0 [e^{-\kappa x} \sin(\omega t - \kappa x) - e^{-\kappa(x+2a)} \sin\{\omega t - \kappa(x+2a)\}] \\ &= A\theta h_0 e^{-\kappa x} \sin(\omega t - \kappa x + \delta) \\ \kappa &= \sqrt{\frac{\omega S}{2T}} \end{aligned}\right\} \qquad (5)$$

Tidal effects transmitted landwards through the aquifer of great extent under the sea bottom is approximated as follows, taking the value of a is large enough in equ. (5).

$$h = \frac{\theta}{2} h_0 e^{-\kappa x} \sin(\omega t - \kappa x) \qquad (6)$$

It is found that the amplitude of the piezometric level in this case is about half compared with the result simply applied the term of Jacob's tidal efficiency.

A good example applicable equ. (5) is provided by analysing the tidal effect on the confined aquifer in the coastal region in Aichi Prefecture, Japan, where the land has subsided and the salt damage has progressed through the groundwater under water table condition. [Kikkawa, 1956]. There are many irrigation wells tapping the deep confined aquifer which yield comparatively fresh water under 60 mg/1 of Cl′ content, whereas the shallow groundwater is quite contaminated by the sea water. However, the water level in these artesian wells are observed to be always lower than the sea level even in the period of no irrigation. It will be concluded, therefore, that the confined aquifer in this area is not in contact with the sea.

Fluctuations in the piezometric levels are observed at four wells with the ocean tide, a part of which is shown in Fig. 1. The relations among the diminishing rates of amplitude and the time-lags of fluctuation with the intervals of landward distance between each observed well give fair agreements with those expected from the analysis by equ. (4) or (5), and provide the value of S/T in this aquifer as 2×10^{-7} hour/m^2 Harmonic analysis of the observed records at the nearest well to the sea shows that the phase is appeared to be in advance of ocean tide for 0.35 hours, though the amplitude is reduced to 0.14. Such phenomena cannot be explained by the relation of equ. (4).

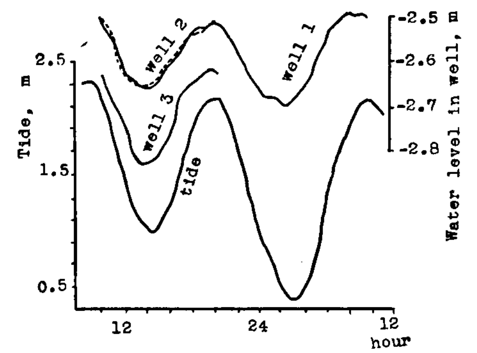

Fig. 1 Water levels in observation wells and ocean tide in Aichi Prefecture in Oct. 1954.

Inserting these informations to equ. (5), it is estimated that $a=2700$ (m) and $\theta=0.36$.

It is also infered that the tidal effects on the piezometric levels with apparent phase-advances, separately reported for a few of coastal artesian wells in Beppu [Nomitu and Seno, 1939] or Asamushi [Maeda, 1936] Hot Spring District in Japan, may originate from the similar mechanism as above-mentioned, in which the ocean tide acts as the variations of the loading weight upon the submarine aquifer.

4. Effects of the ocean tide on the leaky aquifer in Beppu City

Beppu is one of the large hot spring districts in Japan. The main part of the city has the area of about 4 km^2 situated on the coastal alluvial plain and includes over seven hundred artificial wells, each of which always yields the thermal water of about 10~20 l/min on average. Main aquifer is considered as artesian type stratified under the ground from about 20 m to over 400 m depths, though the bottom cannot be ascertained up to today. Fig. 2 indicates the distribution of the heights of piezometric surface with reference to the mean sea level. The height was observed separately for each well artificially stopping the discharge, during which conditions of flow out through other wells were kept as usual as possible. [Seno and Kikkawa, 1961].

It is possible for general state to take a hydrological model of subterranean thermal water in this area as the groundwater flowing through only one confined

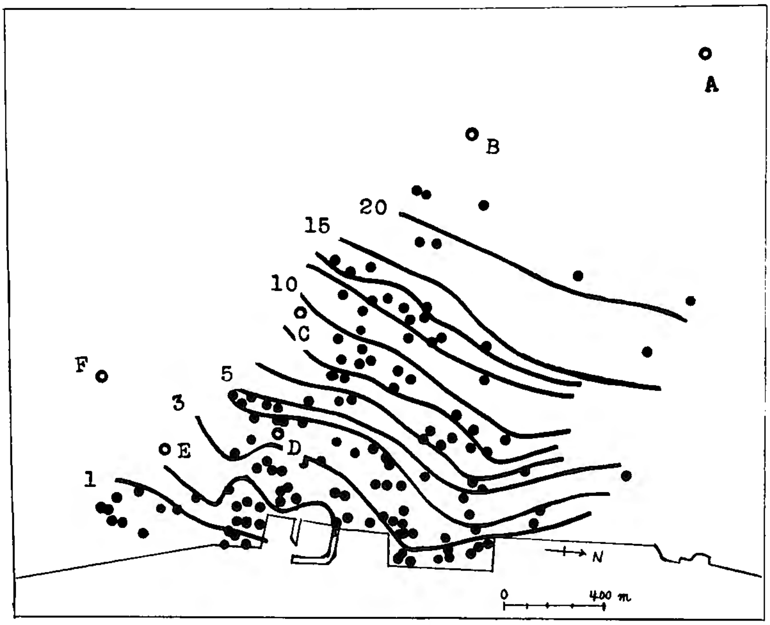

Fig. 2 Distribution of the piezometric level in main part of Beppu City. The heights are expressed by meter from the mean sea level. Dots indicate the positions of observation wells of the piezometric surface. Circles are the positions of the observation of Earth tide by Nishimura.

aquifer, because the distribution of temperature, piezometric level or chemical components observed for each well indicates the fair mixture of the water between different depths of the aquifer, though local properties can be found in some parts.

Nomitu and Seno [1939] observed the influences of the ocean tide on the flow-rates through many wells distributed over the whole area of the main part of Beppu City and noticed the typical states as follows.

The variation in the flow-rate is in proportion to that in ocean tide and diminishes with distance from the coast. Phase difference between the flow-rate and tide is not regularly appeared with the distance from the coast, though some advances or lags are found within two hours.

Nomitu [1940] introduced a new mechanism for the transmission of tidal effect through the aquifer tapped by multiple flowing wells. Substituting the effect of leakage from the aquifer to the ground surface, uniformly distributed on the whole aquifer, for that of flows through multiple wells, he gave the analytical method, the solution of which was fairly applied to the observed tidal influences in Beppu.

$$T\frac{\partial^2 h}{\partial x^2}=bh \qquad (7)$$

$$h=h_0 \sin \omega t \qquad \text{at} \quad x=0$$

Final is as follows.

$$h=h_0 e^{-\sqrt{\frac{b}{T}}x} \sin \omega t \tag{8}$$

where b is called as the coefficient of leakage. The value of diminishing factor $\sqrt{b/T}$ is estimated as 5×10^{-3} (m^{-1}) from the observed results shown in Fig. 3.

Some additions to the properties expressed in equ. (7) were attempted in order to investigate the validity of Nomitu's mechanism. Equ. (7) was developed to two-dimensional flow and used to treat the practical problem predicting the effects by future withdrawals of thermal water. [Kikkawa, 1959].

Kozaburo Yamashita [1961] carried out many pumping tests over this area and found the interesting informations on the coefficient of leakage. That is

$$b=b_1+b_2$$
$$b_1/T=1.2\,N\ (m^{-2}), \quad b_2/T=2.84\times10^{-4}\ (m^{-2})$$

where N is the density of the distribution of wells around the pumped well designated by numbers of well in unit area. He called b_1 as the coefficient of outer leakage, concerned with only the flow out through artificial wells and b_2 as that of inner leakage, originating from the natural effluent or influent through semi-impermeable layers confining the aquifer tapped by test wells.

It is noticed that the value of b/T derived from the results for tidal influences is much smaller than that from pumping tests. When we treat the geohydrological problem in Beppu, of comparatively small scale, for example, the effect of pumping test for some hours, it will be a possible approximation to replace the three-dimensional problem among wells, partially penetrating the whole stratified aquifer, by a simplified model of one-dimensional flow through an aquifer, separated by semi-confining layers, and to take account of the effect of inner leakage from neighboring aquifers, in which the water pressures are assumed being kept constant in the period of the test. But, on the contrary, as to the problem over the large area, for example, to the tidal influence on the piezometric level, the difference of fluctuations in water pressure between each separated aquifer becomes smaller than that for the above-mentioned case, and the better approximation may be expected by assuming that the tidal effect is propagated uniformly through the whole alluvial strata, as if it is one confined aquifer. If we represent the coefficients of leakage as b' and b and those of transmissibility as T' and T respectively for pumping test and tidal influence, it is possible to consider by rough approximation that $b<b'$, because the inner leakage for the tidal influence may occur only to the shallow aquifer under water table condition through which tidal effect is rapidly diminished,

and that $T'<T$, because T' is the value of transmissibility for only a part of the whole stratified aquifer. Considering that the more accurate analysis is quite difficult without three-dimensional complicated treatment, we only expect that the value of b/T in the simplified one-dimensional analysis for the tidal effect may be much smaller than found by the pumping tests.

The other problem to analyse the tidal effect in Beppu is to find the hydrological condition at the submarine boundary. Though the sea-water encroachment has clearly recognized in the southern part, we can take still fresh thermal water in other coastal parts even through the wells tapping over 200 m depths, where the zone of salt water would be expected from a point of view of density current of sea water through the boundary of the aquifer. [Seno and Kikkawa, 1959]. It suggests the closed condition between the main aquifer and the sea, similarly as treated in the last section. However, we can meet with the seepage of thermal water into the sand beach in this region during the period of ebb-tide. It is, then, assumed that the thermal groundwater under artesian condition is not directly in contact but in indirect contact with the ocean, leaking out through the upper semi-impermeable stratum. In such a case, sea-water intrusion is not possible so far as the pressure of artesian water is higher than that of sea water at the top of the aquifer. If so, the next equation may give more general informations than equ. (7) for the tidal effect on the thermal groundwater in Beppu.

$$\frac{\partial(h-\theta p)}{\partial t}=\frac{T}{S}\frac{\partial^2 h}{\partial x^2}-b_1 h-b_2(h-p) \tag{10}$$

where b_1 is the coefficient of outer leakage through artificial wells and b_2 is that of inner leakage through upper semi-confining stratum. The representation of p and the boundary condition are similar as those in the last section.

$$0\geqq x\geqq -a \ : \ p=h_0 \sin \omega t, \quad x>0 \ : \ p=0$$

$$\frac{\partial h}{\partial x}=0 \quad \text{at} \quad x=-a$$

The final solution satisfied in the region $x\geqq 0$ is as follows.

$$h=\frac{h_0}{2}\frac{\sqrt{b_2^2+\theta^2 S^2\omega^2}}{\sqrt{b^2+S^2\omega^2}}\left[e^{-\sqrt{\frac{\sqrt{b^2+S^2\omega+b}}{2T}}x}\cdot\sin\left\{\omega t-\sqrt{\frac{\sqrt{b^2+S^2\omega^2}-b}{2T}}x-\tan^{-1}\frac{S\omega}{b}\right.\right.$$

$$\left.+\tan^{-1}\frac{\theta S\omega}{b_2}\right\}-e^{\sqrt{\frac{\sqrt{b^2+S^2\omega^2}+b}{2T}}(x+2a)}\cdot\sin\left\{\omega t-\sqrt{\frac{\sqrt{b^2+S^2\omega^2}-b}{2T}}(x+2a)-\tan^{-1}\frac{S\omega}{b}\right.$$

$$\left.\left.+\tan^{-1}\frac{\theta S\omega}{b_2}\right\}\right]$$

where $b=b_1+b_2$.

Since $\sqrt{b^2+S^2\omega^2}=b\left(1+\frac{1}{2}\frac{S^2\omega^2}{b^2}+\cdots\cdots\cdots\right)$

and $\tan^{-1}\frac{S\omega}{b}=\frac{S\omega}{b}-\frac{S^3\omega^3}{3b^3}+\cdots\cdots\cdots$,

it is simplified as close approximation, when $b \gg S\omega$ and $b_2 \gg \theta S\omega$.

$$h=\frac{h_0}{2}\frac{b_2}{b}\left(1-e^{-\sqrt{\frac{b}{T}}2a}\right)e^{-\sqrt{\frac{b}{T}}x}\cdot\sin\left(\omega t-\frac{S\omega}{b}+\frac{\theta S\omega}{b_2}\right)$$

Setting B for the reduction factor, further approximation leads to the next simple relation.

$$\left.\begin{aligned} h &\doteqdot Bh_0 e^{-\sqrt{\frac{b}{T}}x}\cdot\sin\omega t \\ B &= \frac{b_2}{2b}\left(1-e^{-\sqrt{\frac{b}{T}}2a}\right) \end{aligned}\right\} \qquad (11)$$

This approximate solution shows the similar property of tidal effect, diminishing with the distance from the coast, as shown in equ. (8).

Nomitu, Seno and Kaoru Yamashita [1940] reported that the variation in the flow-rate, Q, is proportional to that of the piezometric surface at each well in Beppu, even under unsteady condition, so far as the range of variation in flow-rate is not so large.

$$Q=cAh$$

where c is a constant specified for each well and A is a crossectional area of observed well. The mean value of c was taken as 20.0 (min^{-1}) among 40 wells in Beppu by Seno [1940] and as 19.7 (min^{-1}) among 58 wells in the same area by Seno and Kikkawa [1961].

Combining the above relation with equ. (11), it follows that

$$\frac{Q}{Ah_0}=cBe^{-\sqrt{\frac{b}{T}}x}\cdot\sin\omega t \qquad (12)$$

Plotting the logarithmic values of the amplitude of Q/Ah_0 observed by Nomitu and Seno [1939] with the distances from the coast, we obtain Fig. 3, on which plotted points are considerably scattered but give the distribution around an approximate linear relation. Considering that the scattering of the graph may originate chiefly from that of the value of c for each observed well, we take the linear relation as passing the central position of the graph and find the next values to satisfy the equ. (12).

$$\sqrt{b/T}=5\times10^{-3} \quad (\mathrm{m}^{-1})$$
$$cB=3.7 \quad (\mathrm{min}^{-1})$$

If we take the mean value of c as 20, the factor of reduction is given as the next value.

$$B=0.19$$

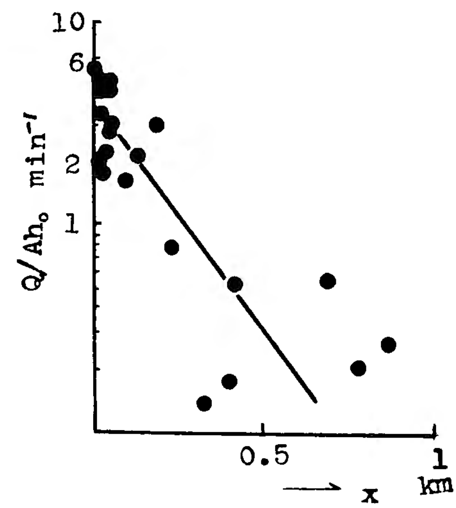

Fig. 3 Graph showing the relation between the tidal effect on flow-rate at each well and the distance from the coast, observed in Beppu by Nomitu and Seno.

5. Effect of the tidal fluctuations in groundwater pressure on the tilting motion of the ground

It is a well-known fact that, among natural effects producing the tilting motion of ground surface observed at a station close to the sea, the bending of the earth surface caused by the alternate tidal load of neighboring sea water is most important. Nishimura [1950] observed the Earth tide with tilt-meters at six stations being several meters below the ground surface in Beppu, the positions of which are indicated in Fig. 2, and found the strange results, contrary to abovementioned expectancy. He subtracted all commonly known effects of the Moon from the observed data and expressed the residuals as anomalies of Earth tide in Beppu, which were explained as phenomena owing to the geologic structure under the influence of an active fault running through the southern part of Beppu City.

The most interesting property of those anomalies is that they show landward tiltings about at the high tide of Beppu Bay as if the earth surface is raised along the coast line with the sea level. In order to make clear such informations, elements of westward components of anomalies reported by Nishimura are rewritten on the next table, in which amplitude is indicated by radian of gradient for half range of tide, being 1 m, and phase is expressed by lag in lunar hour to the tide of Beppu Bay. The direction of E-W is almost perpendicular to the coast line. Logarithmic values of amplitude are plotted against the distances from the coast in Fig. 4, and show almost linear relation, especially fairly satisfied to the five stations besides A, furthest from the coast.

Letting H designate the anomalous rise of earth surface with reference to its mean stage, the next equation is adopted to satisfy the relation, shown in Fig. 4,

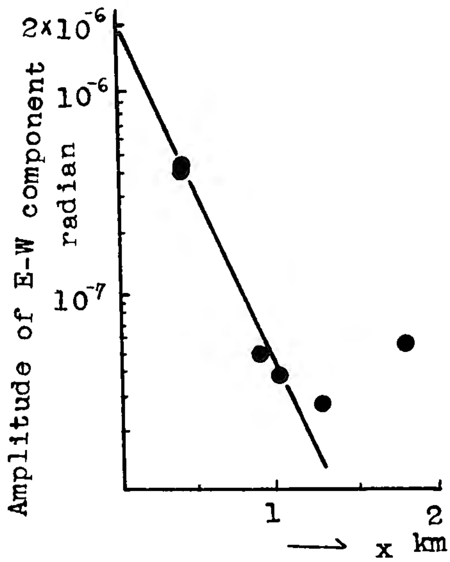

Fig. 4 Westward anomalies of Earth tide with the disatances from the coast, observed in Beppu by Nishimura.

Table 1 *E-W* components of anomalies of Earth tide in Beppu.

Station	Amplitude (radian)	Phase (lunar hour)	Distance from the coast (m)
A	5.43×10^{-8}	1.7	1800
B	2.70	1.	1300
C	4.66	−0.3	900
D	41.5	1.	400
E	39.1	0.5	400
F	3.73	1.	1000

between the ocean tide and the anomalous tilting of the ground along x axis, if it is possible to neglect the difference of phase within two hours.

$$\left.\begin{aligned}\frac{\partial H}{\partial x}&=-\beta h_0 e^{-\kappa x}\cdot\sin\omega t\\ \beta&=2\times10^{-6}\quad(\mathrm{m}^{-1})\\ \kappa&=4\times10^{-3}\quad(\mathrm{m}^{-1})\end{aligned}\right\}\tag{13}$$

On the other hand, we have already found the mathematical evaluation of the tidal influence on piezometric level in Beppu as equ. (11) within the error of two hours of phase difference. If it is possible to presume that the elastic change in aquifer thickness is perfectly reflected by rise or fall of ground surface, the variation of ground tilting, represented by $\frac{\partial H'}{\partial x}$, must be expected to follow the tidal variations in the pressure of confined groundwater.

$$\frac{\partial H'}{\partial x}=S'\frac{\partial h}{\partial x}=-BS'\sqrt{\frac{b}{T}}\,h_0 e^{-\sqrt{\frac{b}{T}}x}\cdot\sin\omega t \qquad (14)$$

It is of interest to note that the formulation of observed anomalous tilting, $\frac{\partial H}{\partial x}$, expressed by equ. (13) is quite similar to that of the presumed tilting, $\frac{\partial H'}{\partial x}$, in equ. (14). From a quantitative point of view, the difference between the values of κ in equ. (13) and $\sqrt{b/T}$ in equ. (14) or (12) is so small compared with the possible errors in the course of analysis for observed data that it is possible, as close approximation, to take equs. (13) and (14) identical with each other. It thus appears that the main part in the *E-W* components of observed anomalies of Earth tide in Beppu shows the reflection of the variations in the thickness of artesian aquifer, being almost in proportion to the tidal changes in the ground-water pressure. Moreover, it will be expected that observations on the tilting motion of earth surface in other coastal plains have possibilities to involve the effects of ocean tide transmitted through the ground-water in the artesian aquifer, which affect in the opposite direction to the effects of tidal loading on the ground.

Another point of interest in a view of geohydrology is that data for Earth tide gave the direct support to the assumption for the elastic property of the aquifer, which follows the Young's law under the effect of the semi-diurnal fluctuations in the pressure of confind water. The value of S' of the aquifer in Beppu is then estimated as follows, inserting the value of β in equ. (13) and those of B and $\sqrt{b/T}$ already found in the last section.

$$S'=\frac{\beta}{B\sqrt{b/T}}=2\times10^{-3}$$

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